

Study on Principal-agent Theory

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Abstract. The core of principal-agent theory is to solve incentive problem of the principal to the agent when the conflict of interest and information asymmetry. This paper analyzes traditional bilateral principal-agent theory with research purpose, basic assumptions, model establishment, solution and conclusion as basic framework, and summarizes three extended forms of principal-agent theory.

1. Introduction

In the Arrow-Debreu model, the manufacturer is seen as a black box, which absorbs various factors to invest and adopts profit maximization under budget constraints. But the model ignores information asymmetry and incentives within the enterprise. Beginning in the late 1960s and early 1970s, a group of economists went deep into the internal relations of enterprises and created principal-agent theory. Famous founders of the theory include Wilson, Ross, Mirrlees and Holmstrom. For more than 40 years, the principal-agent model has developed relatively maturely, which has extended multi-agent theory, common agent theory and multitasking agency theory from the traditional bilateral principal-agent theory. The following is a brief review of principal-agent theory from basic framework and extended forms.

2. The basic framework of principal-agent theory

Principal-agent theory begins with the discussion of bilateral principal-agent problems which have single principal, single agent, and single agency affairs. It is the most abstract and ideal form of all principal-agent problems. It is also the basis of other principal-agent theories.

2.1 Research purpose

In order to maximize interests, the principal grants the decision-making power of their own resources to the agent, and requires the agent to provide services or actions that are beneficial to interests of the principal. However, agents also pursue their own utility maximization. In the case of inconsistent interests and information asymmetry, moral hazard will follow. The agent may be tempted to exercise the power of decision-making power granted by the principal, and use the advantage of information to damage interests of the principal, thus creating an agency problem.

It is precisely because of the existence of agency problem that the principal must establish an effective contract to constrain and motivate the agent's behavior, which can solve agency problem, reduce agency cost, improve the efficiency of the principal agent, and better realize the maximization of self-interest.

2.2 Basic assumptions

There are two participants. The principal is risk neutral and the agent is risk avoider. Both follow the economic man assumption.

Principal-agent theory assumes that information between the principal and the agent is asymmetric. In other words, the principal does not observe the agent's actions, nor can he judge the actions taken by the agent based on business results. α refers to the degree of the agent's effort, such as the manager's business behavior. The client is in an information disadvantage. θ is a random variable and indicates natural state, such as market changes. x represents profit, which is determined by a combination of α and θ , that is, $x = x(\alpha, \theta)$.

Principal-agent theory assumes that the interests of the principal and the agent are inconsistent. The principal wants the agent to work hard, but the agent hates the work. The principal's utility function is $V=V[x-s(x)]$. The agent's utility function is $U=U[s(x)]-c(\alpha)$, where $s(x)$ indicates the remuneration paid by the principal, and $c(\alpha)$ represents the negative utility of the agent's efforts. U_0 refers to the utility of the agent in other firms, also known as retained utility.

2.3 Model establishment

With the above assumptions, the distribution function parameter method is used to convert the distribution function of the natural state θ to the output x under the effort level α . In this way, a proxy model can be established.

First, principal-agent theory considers maximizing the expected utility of the principal.

$$\max_{s(x), \alpha} \int V[x-s(x)]f(x, \alpha)dx \quad (1)$$

In formula (1), $f(x, \alpha)$ is a density function.

It can be seen from formula (1) that under full information conditions, the principal can directly adopt a mandatory contract, specifying how much action the agent must take. In other words, the principal can directly choose a reasonable α and $s(x)$ to maximize the formula (1). However, considering the information asymmetry, the principal cannot observe the agent's actions, that is, the agent's specific action α cannot be specified. The principal's optimization problem is faced with information constraints.

Second, principal-agent theory considers how the client's information constraints are modeled.

In order to execute the contract, the principal must consider interests of the agent. That is to say, the principal expects the agent's effort α to maximize the agent's expected utility, which is the incentive compatibility constraint. In addition, the reason why the agent is willing to pay α of the principal's expectations is that the expected utility of the agent's efforts is not less than the expected utility level under other efforts, which is the participation constraint.

Incentive compatibility constraint:

$$\alpha \in \arg \max \int \{U[s(x)]-c(\alpha)\}f(x, \alpha)dx \quad (2)$$

Participation constraint:

$$\int \{U[s(x)]-c(\alpha)\}f(x, \alpha)dx \geq U_0 \quad (3)$$

2.4 Model Solution

To simplify the model, we assume that there are only two actions in the agent's strategic space: α_H (high effort level) and α_L (low effort level), and the corresponding density functions are $f_H(x)$ and $f_L(x)$. From a practical point of view, when x is small, the probability that the agent is lazy is large. So $f_H(x)$ is the first-order random occupation better than $f_L(x)$, that is, $f_H(x) < f_L(x)$.

The principal obviously wants the agent to take a high effort level α_H , so the corresponding constraints are changed as follows:

$$\int \{U[s(x)]-c(\alpha_H)\}f_H(x)dx \geq \int \{U[s(x)]-c(\alpha_L)\}f_L(x)dx \quad (4)$$

$$\int \{U[s(x)] - c(\alpha_H)\} f_H(x) dx \geq U_0 \quad (5)$$

By establishing a Lagrangian function, the first-order method is used to obtain the conditions that the optimal compensation contract must satisfy:

$$\frac{1}{U'[s(x)]} = \lambda + \mu \left[1 - \frac{f_L(x)}{f_H(x)} \right] \quad (6)$$

In formula (6), λ , μ respectively are the multipliers of formula (5) and (4).

2.5 Model conclusion

First, according to Borch, it can be known that when $\mu=0$, equation (6) becomes the optimal risk sharing condition. At this time the solution is optimal, in other words, the right side of equation (6) is a constant, which means the agent's remuneration is fixed. The agent does not bear any risks, and all risks are borne by the client. However, according to Holmstrom, equation (6) cannot be a constant, because the reward paid by the principal is related to the profit x . The principal will take incentives to make the agent bear certain risks, so equation (6) deviates from the optimal risk-taking. The result is sub-optimal, which cannot achieve Pareto efficiency in the case of information symmetry.

Second, we further assume that γ represents the prior probability when the agent works hard, then the posterior probability is $\gamma(x)$. Equation (6) is rewritten according to Bayes' rule as:

$$\frac{1}{U'[s(x)]} = \lambda + \mu \left[\frac{\gamma(x) - \gamma}{\gamma(x)(1 - \gamma)} \right] \quad (7)$$

Equation (7) shows that when the posterior probability is less than the prior probability, $s(x)$ is reduced. This means that when the profit is a certain value x , causing the principal's posterior probability based on the result is less than the prior probability, the principal obviously reduces the agent's remuneration. It is consistent with actual situation.

3. The extended form of principal-agent theory

Bilateral principal-agent theory is the most abstract and ideal form, but its explanatory power to reality is limited. With the development of economy and society, scholars continue to expand the principal-agent theory, thus developing multi-agent theory, common agent theory, and multitasking agency theory.

3.1 Multi-agent theory

Different from the bilateral principal-agent theory, there are multiple agents in the multi-agent theory. There are mutual influences between the agents, and the outputs can be compared with each other. The theory of multi-agent believes that the principal is not obliged to observe the behavior of the agent, but the observation cost is too high. However, when there are multiple agents, the behavior of one agent can provide information of other agents, leading to the fact that the cost of information observing the behavior of agents is not so high. Therefore, when the output of an agent is related to the actions of other agents, the optimal incentive contract can determine the payment to the agent based on the output of the other agent.

3.2 Common agent theory

Bernheim and Whinston proposed a common agency theory when multiple manufacturers jointly commission an agent for production in actual economic activity. Different from the bilateral principal-agent theory, there are multiple principals in common agent theory. Since multiple principals are involved, the core of the research on common agency theory is not only how to achieve efficient output, but also how to distribute production profits between multiple principals and single agents.

The theory of common agent mainly has three aspects of development research. First, Martimort expanded the principal-agent theory from complete information to incomplete information, from the homogeneity of the principal task to the heterogeneity, from static to dynamic, and gradually moves the model closer to reality. Second, Mezzetti studied the effectiveness of incentive clauses and the design of incentive mechanisms under the common agency framework. Third, the common agent theory is applied to other fields. For example, Parigi applied the common agency theory to financial contracting.

3.3 Multitasking agency theory

Holmstrom and Milgrom argued in 1991 in an article entitled “Multitasking Principal-Agent Issues: Incentive Contracts, Asset Ownership, and Job Design”. They thought bilateral principal-agent theory is too abstract and difficult to use to analyze a wide range of economic organizational issues, such as, asset ownership, job design, and authoritative distribution within the organization, and so on. So they proposed a multitasking agency theory. Different from the bilateral principal-agent theory, the multitasking agency theory involves the agent performing multiple tasks entrusted by the principal at the same time, which makes the problem very complicated. For the agent, he has to make decisions on the effort level of each task, which makes the agent's participation constraint complicated. The minimum utility of the agent is no longer equal to the retained utility U_0 , but the minimum utility combination of the agent's effort to participate in multiple tasks. For the client, he needs to design optimal contract incentives for each task and the level of effort required by the agent, which translates the original single incentive compatibility constraint into multiple partial or holistic forms.

The research on multitasking agency model is mainly divided into two categories. On the one hand, scholars study the design of incentive mechanisms and the ways to reduce agency costs when there are alternative or complementary effects between different work tasks. On the other hand, they study the design of incentives and the ways to reduce agency costs when there are multiple performance indicators.

4. Summary

This paper mainly expounds the theory of bilateral principal-agent, and compares the differences between multi-agent theory, common agent theory, and multitasking agency theory. The new model is more explanatory because it is closer to reality. Therefore, the basic framework of bilateral principal-agent theory can be applied to other principal-agent theories to analyze various problems in corporate governance in China.

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